Microstrip Design Laboratory

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***Abstract*—Design concepts, fabrication, measurements, and optimization of a microstrip filter, directional coupler, and amplifier are presented. The microstrip circuit implementation is simple and easily alterable for optimization. Results from an undergraduate laboratory course are presented which show reasonable agreement with design parameters, and the optimization process adds to the educational experience.**

I. Introduction

Microwave undergraduate laboratories have traditionally emphasized basic electromagnetic theory and microwave measurements. Most of the experiments deal with TDR, slotted-lines, and microwave tubes including power, frequency, and phase measurements.

The proliferation of microstrip circuit technology in industry begs for student exposure in this area. Such exposure, especially when a simple macro-level implementation that is conveniently alterable is used, provides the additional benefit of being an introduction to microwave integrated circuits (MIC's). A byproduct has been that students come away with the comforting and satisfying feeling that microstrip transmission lines really work.

II. Design Concepts

A combination of design equations and the Smith chart constitute the basic design tools. Computer-aided engineering software is often used for verifying the design. Of course, the CAE tools could be used initially as in industry, however, this would preclude the insight gained from working the equations and graphical aids. Thus, expediency is somewhat sacrificed for learning. It is hopeful, of course, that the design equations are derived in the lecture portion of the course, but some are largely empirical in any case. Since microstrip transmission lines are used in all the circuits reported here, the design procedure for microstrip is presented first.

A. Microstrip Design

The ease of manufacture of microstrip transmission line has made microstrip line of major importance for microwave circuits, especially at the lower microwave frequencies. The fundamental principle for determining the characteristic impedance of a transmission line from the electrostatic relationship is

$$Z_o = \frac{1}{v_p C} \tag{1}$$

where C is the line capacitance/unit-length and v_p, the velocity of propagation, is given by

$$v_p = c/\sqrt{\epsilon_{r,\text{eff}}} \tag{2}$$

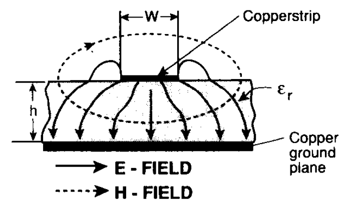

Fig. 1. End view of microstrip transmission line with electromagnetic field intensity lines superimposed.

where $\epsilon_{r,\text{eff}}$ is the effective relative dielectric constant, or permittivity, of the dielectric material through which the electromagnetic (EM) field propagates, and c = 3E8 m/s. As illustrated in Fig. 1, the electric field emanates from the conducting strip of width W, passes through a layer of air (ϵ_{r_o} = 1) and then down through a solid dielectric (ϵ_r) sheet of thickness, or height, h to terminate on the ground-plane conductor.

Because of the discontinuity of the EM field at the air-solid dielectric interface, quasi-TEM propagation (and dispersion) results, and a set of largely empirical equations must be solved for calculating the effective dielectric constant as well as the W/h ratio required to achieve a desired characteristic microstrip line impedance Z_o; see Fig. 2. For best accuracy, two sets of equations are used for both Z_o and $\epsilon_{r,\text{eff}}$ at frequencies below which dispersion may be neglected. The critical frequency is given by [1]

$$f_{\text{GHz}} = 0.3\sqrt{\frac{Z_o}{h\sqrt{\epsilon_r - 1}}} \tag{3}$$

where h is in centimeters.

To compute W/h assuming the strip conductor thickness is negligible ($t/h \le 0.005$), use one of the following equations [2] based on a break point at $W/h = 2$:
For $W/h \le 2$:

$$\frac{W}{h} = \frac{8e^A}{e^{2A} - 2} \tag{4}$$

where

$$A = \frac{Z_o}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right) \tag{5}$$

or, for $W/h \ge 2$:

$$\frac{W}{h} = \frac{2}{\pi}\left\{B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r}\left(\ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r}\right)\right\} \tag{6}$$

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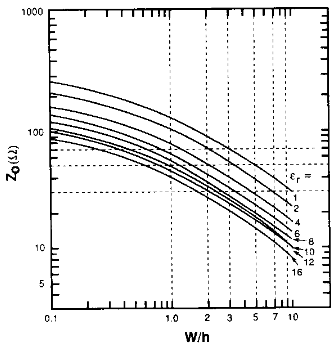

Fig. 2. Characteristic impedance versus microstrip line width-to-height ratio and board dielectric constant.

where

$$B = 592.2/(Z_o\sqrt{\epsilon_r}). \tag{7}$$

To compute the effective dielectric constant for negligible strip conductor thickness ($t/h < 0.005$), use one of the following equations [3]:
For $W/h \leq 1$:

$$\epsilon_{r,\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left[(1 + 12h/W)^{-1/2} + (1 - W/h)^2/25\right] \tag{8}$$

or, for $W/h \geq 1$:

$$\epsilon_{r,\text{eff}} = 0.5\left[(\epsilon_r + 1) + (\epsilon_r - 1)(1 + 12h/W)^{-1/2}\right]. \tag{9}$$

The value of h is determined by the availability of suitable microwave circuit board material. Common values of h for RT/duroid (Rogers Corp., Chandler, AZ) are 31 mils and 62 mils (1.575 mm), and $2 \leq \epsilon_r \leq 2.6$. For most applications, (8) and (9) are accurate for computing the propagation wavelength from

$$\lambda_g = \frac{c}{f\sqrt{\epsilon_{r,\text{eff}}}} \tag{10}$$

since quasi-TEM propagation is assumed.

B. Low-Pass Filter

Microstrip filters can get quite elaborate, so a simple low-pass filter is a good place to start; perhaps even before mundane experiments such as 50 Ω microstrip transmission lines connected to various loads. Even making reflection coefficient measurements seems more interesting after seeing that a few strips of copper can produce a passive device with transmission characteristics studied in previous courses, and one that is vital to communication and other systems.

For an n-pole Butterworth low-pass filter (LPF), the kth normalized component value g_k is computed with [4], [5]

$$g_k = 2\sin\left[\pi(2k - 1)/2n\right] \tag{11}$$

from which capacitance is computed from

$$C_k = g_k/(\omega_c R) \tag{12}$$

or inductance from

$$L_k = g_k R/\omega_c \tag{13}$$

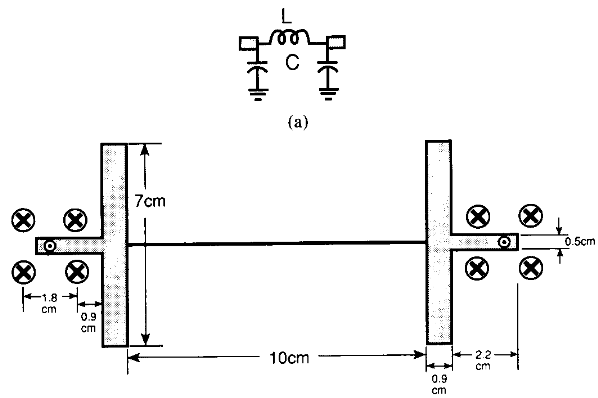

Fig. 3. (a) Microstrip low-pass filter layout including 50 Ω lines-to-ground-plane-side type-N connectors. (b) The circuit schematic.

where ω_c is the 3 dB cutoff frequency. A pi-section filter will have a shunt capacitor C_1, followed by a series inductor L_2, followed by a shunt capacitor C_3. Of course, a single pi-section filter will require $C_3 = C_1$ assuming that both source and load have a characteristic impedance equal to R. Equations for Chebyshev realizations are also found in [4], [5].

Circuit implementation of a three-pole LPF in microstrip is illustrated in Fig. 3. Fig. 3(a) shows the effective schematic, and the copper strip layout is illustrated in Fig. 3(b).

The input lines have 50 or 75 Ω characteristic impedance. Capacitors consist of low Z_o balanced, open-circuited tuning stubs, and the series inductors are high Z_o microstrip lines effectively short-circuited or, if necessary at frequencies below about 1 GHz, a short length of wire.

Each circuit capacitor is a balanced stub. That is, each capacitor consists of two single-ended, open-circuited stubs connected in parallel at the filter centerline. The capacitive reactance X_c is computed using (11) and (12), and Z_{in} looking into one of the open-circuited stubs must be designed to have a value of

$$Z_{\text{in}} = 2X_c \tag{14}$$

at the specified filter 3 dB cutoff frequency.

From transmission line theory for an open-circuited stub,

$$Z_{\text{in}} = -jZ_o \cot\Theta \tag{15}$$

where the stub length is found from

$$\Theta = \beta\ell = \frac{2\pi}{\lambda_g}\ell \tag{16}$$

and the guide wavelength, λ_g, is computed from (10). Since $\cot\Theta = 1/\tan\Theta$,

$$\Theta = \tan^{-1}\frac{Z_o}{2X_c} \tag{17}$$

where $Z_o \leq 30$ Ω for practical, high-Q, low characteristic impedance microstrip line. Hence, the full length of the copper strip is 2ℓ after solving (17) and then (16), and the width W is computed from (4) or (6).

The inductor is implemented either with a high Z_o (about 120 Ω) microstrip line, or perhaps a wire connected to the center of both the filter input balanced-stub capacitor and the output balanced-stub capacitor. The low Z_o capacitors act as short-circuits and, therefore, the required microstrip line length is computed from (16) and

$$X_L = jZ_o \tan \Theta. \tag{18}$$

Because of the high Z_o, W/h is computed from (4) but the value should not be less than 0.5. If W becomes impractically narrow, approaching 1 mm, use a length ℓ_L of wire of diameter D which has inductance on nonferromagnetic circuit boards, easily derived using image theory, as

$$L = \frac{\mu_o \ell_L}{2\pi} \ln \frac{4h}{D} \tag{19}$$

in henrys where ℓ_L is in meters.

C. Amplifier Design

Microwave devices are characterized by S-parameters at a given frequency. Variations with frequency are usually shown on Smith chart plots in the data sheets for these devices. Transistors are characterized as two-port devices because they are almost always used in common emitter amplifiers so that the emitter lead or ribbons are grounded and are therefore common to the input (base) and the output (collector) ports.

The gain of a single-stage amplifier can be analyzed by considering the power gain G_o of the active device (transistor) connected directly to a 50 Ω (Z_o) generator and load, G_1 the gain (or loss) provided by an output matching network, and G_2 the gain (or loss) provided by a matching network between the active device output and the load.

If input and output matching networks provide complex conjugate matches between transistor and the generator and load, then the transducer power gain, defined as the ratio of power delivered to the load to source available power, is maximum and for a unilateral transistor ($S_{12} = 0$), the maximum transducer power gain is given in [1]

$$G_{\max} = G_1(\max)\, G_o G_2(\max) \tag{20A}$$

$$= \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}. \tag{20B}$$

The matching network reflection coefficients at the input and output are Γ_1, and Γ_2, respectively. If $\Gamma_1 \neq S_{11}$ and $\Gamma_2 \neq S_{22}$, then less than maximum power gain results, given by

$$G = \frac{1 - |\Gamma_1|^2}{|1 - S_{11}\Gamma_1|^2} |S_{21}|^2 \frac{1 - |\Gamma_2|^2}{|1 - S_{22}\Gamma_2|^2} \tag{21}$$

Thus, the gain provided by the ith port matching network is

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2} \quad (i = 1, \text{ or } 2). \tag{22}$$

The most straightforward lab experience is to use the Smith chart to determine the lengths of line and stub to provide single-stub tuning at the input and output. The input stub is placed at the input connector and in combination with a length d_{in} of line to the amplifying device, transforms the source impedance to the complex conjugate of the active device input admittance. Input bias is connected via the ac shorted end of the stub or, if the shortest length of stub is an open-circuit, a quarter-wavelength transformer is attached near the active device input lead. The latter technique is illustrated in Fig. 4.

Also illustrated in Fig. 4 is the output single-stub tuner with a short-circuited stub and 50 Ω microstrip line (length d_o) used for transforming the output 50 Ω load to the transistor output admittance complex conjugate y_o^*. Collector bias is applied at the ac shorted end of the stub, and a choke L improves ac isolation from the power supply. The amplifiers designed and built in the past three years using a Motorola MRF 901 ($f_T = 4.5$ GHz) have been unconditionally stable. The S-parameters for the MRF901 (flatpack with dual-emitter ribbon leads) at 500 MHz, $V_{cc} = 6$ V and $I_c = 10$ mA are: $S_{11} = 0.46\,\underline{/-151^\circ}$, $S_{22} = 0.43\,\underline{/-33^\circ}$, $S_{21} = 7.5\,\underline{/92^\circ}$, and $S_{12} = 0.04\,\underline{/51^\circ}$; angles in degrees. Thus, maximum available gain is 1.03 + 17.5 + 0.89 = 19.4 dB, assuming S_{12} is small enough to be ignored. Actual results vary above and below the value computed.

D. Directional Coupler

Directional couplers are the most difficult microstrip circuits both analytically and in fabrication. Extremely accurate dimensioning is critical at high frequencies for all basic couplers. The most straight forward and adaptable microstrip directional coupler, often called the backward-wave coupler, consists of two quarter-wave microstrip lines placed very close together for edge coupling (or parallel coupling).

The initial analytical development, with even and odd mode impedance analysis and coupling coefficient, is straight forward enough but the electromagnetic coupling problem to derive appropriate line widths and spacing in the nonTEM dispersive microstrip regime still leaves the designer without closed-form solutions. Available nomographs are very limited and do not provide much insight as to how device parameters vary with critical dimensional variations.

An excellent plot of gap spacing and conductor width versus even-mode and odd-mode impedance $Z_{o\text{even}}$ and $Z_{o\text{odd}}$ is provided in Fig. 5 from [6], [7] for $\epsilon_r = 9.6$. Fortunately, this plot can be adapted to other than alumina substrates by realizing that the velocity of propagation, and therefore, microstrip dimensions are inversely proportional to the square root of the effective permittivity. Thus, the design procedure is given as follows: the coupling coefficient k is computed from the desired coupling factor, F(dB), using

$$k = 10^{-F(\text{dB})/20} \tag{23}$$

and then the even and odd mode impedances are found from

$$Z_{o_e} = Z_o \sqrt{(1 + k)/(1 - k)} \tag{24}$$

$$Z_{o_o} = Z_o^2 / Z_{o_e} \tag{25}$$

where Z_o is the system characteristic impedance, typically 50 Ω.

Next the effective permittivity for the microstrip is computed using (9) where W/h is approximately 3 for typical values of $Z_o = 50$ Ω and $\epsilon_r = 2.2$, using (4). For 50 Ω and $\epsilon_r = 9.6$, $\epsilon_{r,\text{eff}} = 6.6$ so both the even and odd mode impedance values computed above can be scaled for a dielectric of $\epsilon_r = 9.6$ by using [8]

$$Z_{o_x}^{[9.6]} = Z_{o_x}^{[\epsilon r]} \sqrt{\epsilon_{r,\text{eff}}/6.6} \tag{26}$$

where $x = e$ for even mode, and $x = o$ for the odd mode impedance. Fig. 5 is then employed, with the value of h for the circuit board used, for determining the gap width (spacing s) and microstrip line width W. Keep in mind that the backward-wave coupler is used for 17 dB or less coupling because the stronger

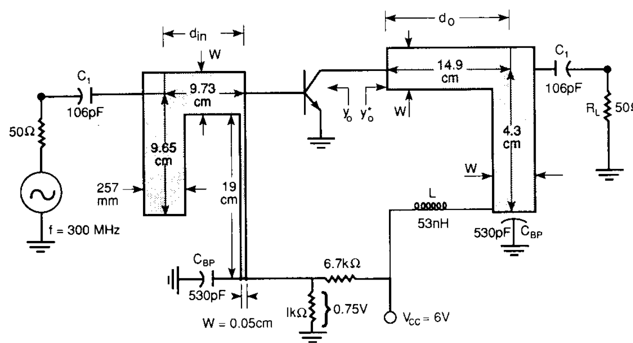

Fig. 4. Microstrip amplifier diagram. Shaded areas represent the copper strip of the microstrip lines.

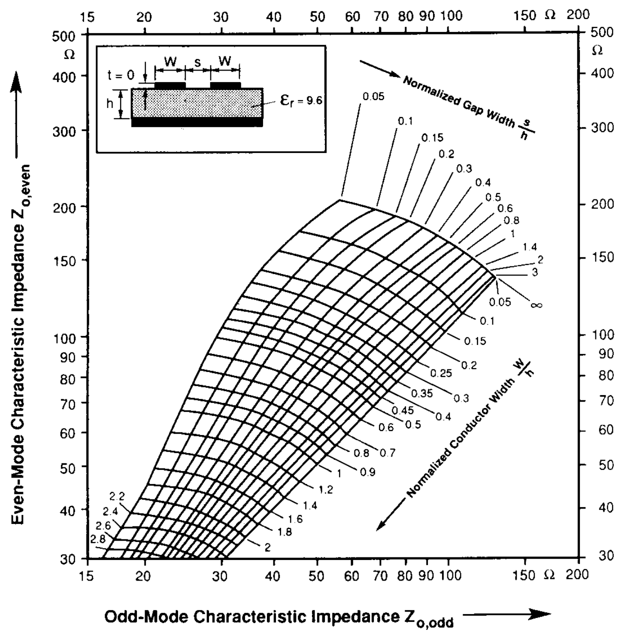

Fig. 5. Backward-wave directional coupler dimensioning for dielectric permittivity of 9.6.

coupling between edge-coupled microstrip lines requires extremely narrow gap spacing and the odd- and even-mode velocities are not approximately equal as required for (24).

III. Fabrication Techniques

The technology involved in the circuit fabrication and the testing depends, of course, on the operational frequencies; not to mention the operational budget. Circuit board dimensions for the filter, directional coupler and the amplifier are within 15 × 12 cm (6 × 4.7 in) at 450 MHz. A branchline coupler was 25 cm (10 in) square!

The most appropriate circuit board material has been the single-side cladded RT/duroid 5870 with $\epsilon_r = 2.33$ and a dielectric thickness of 62 mils (0.244 mm—the thicker the better) graciously provided by Rogers Corporation of Chandler, AZ 85224. The top side circuit track is most conveniently cut from a one-inch width copper tape with single-sided adhesive (Scotch/3M, St. Paul, MN). This allows for quick construction and even quicker alterations. The most straight forward and cost-effective connector arrangement is to drill five holes through the circuit board and bolt-in a male type-N bulkhead connector from the copper-clad ground-plane, then solder the center pin to the circuit-side copper strip. The photograph of Fig. 6 illustrates the construction of the microstrip amplifier. Obviously, a research project or higher operating frequencies will require more elaborate fabrication techniques.

IV. Test Equipment

Realistically, the minimum of test equipment is a sweep generator with detector and oscilloscope. This allows for continuous swept frequency plots of the transmission/gain characteristics of the devices. Using only a CW signal generator will

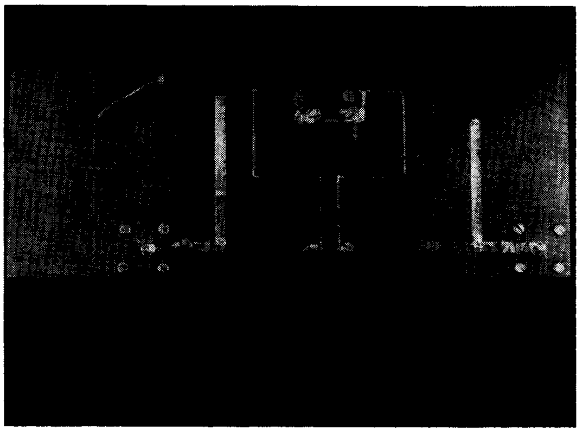

Fig. 6. Microstrip amplifier specimen. Input and output matching is with open-circuit stubs.

require a very patient observer. A marker generator for determining the frequencies at various points on the plot is also important. Of course, resistive summing of the CW "marker" generator with the sweep signal just ahead of the signal detector will provide the "blip" on the scope trace that will calibrate the frequency axis.

Equipment such as the HP 8754A, 4-1300 MHz network analyzer is ideal for measurements at the low microwave frequencies that are convenient and relatively trouble-free. This instrument along with the accompanying HP8502A Transmission/Reflection Test Set allows for normal swept frequency measurements of gain and return loss (for reflection coefficients) as well as polar and Smith chart plots. An old General Radio 1710 RF Network Analyzer proved sufficient for an undergraduate lab but the sweep frequency range is limited to 500 MHz.

V. Results

Fabricated three-pole low-pass filters designed using (11)–(19) had approximately 0.5 dB insertion loss. However, twice the inductor computed length was required in order to achieve our 350 MHz cutoff frequency.

Initially, poor results were achieved by students with the microstrip amplifier so a quiz was given on the amount of reactance and skin-effect resistance that quarter inch (6.35 mm) lead lengths produce at 500 MHz. After students corrected for long lead lengths, amplifier gains were within ± 0.5 dB of design goals. The directional couplers had coupling factors within 1.2 dB of design goals. However, the reflection coefficients and directivities were as high as -15 dB although some were close to -30 dB. The major difficulty was in placing the straight quarter-wavelength lines side-by-side with accurate spacing. We have also had success with an active mixer, an oscillator, and a tapped interdigital three-pole bandpass filter.

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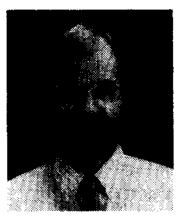

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